Limits of a Functions:

Determine the limit of each function at the indicated value:

1. $$\lim_{t\to 0}\frac{\sqrt{t+9}-3}{t}=\lim_{t\to 0}\frac{\left(\sqrt{t+9}-3\right)}{t}\cdot\frac{\left(\sqrt{t+9}+3\right)}{\left(\sqrt{t+9}+3\right)}=\lim_{t\to 0}\frac{t+9-9}{t\left(\sqrt{t+9}+3\right)}=$$
$$\lim_{t\to 0}\frac{1}{\left(\sqrt{t+9}+3\right)}=\frac{1}{\sqrt{0+9}+3}=\frac{1}{6}$$

2. $\lim_{x\to 0}\frac{\sin x}{x}=1\Rightarrow$ *definition*

3. $\lim_{x\to 0}\sin\frac{\pi}{x}=$ *undefined*

4. $\lim_{x\to 5}\left(2x^2-3x+4\right)=2(5)^2-3(5)+4=50-15+4=39$

5. $\lim_{x\to 1}\left[\sqrt[5]{x^2-x}+\left(x^3+x\right)^9\right]=\left[\sqrt[5]{(1)^2-(1)}+\left((1)^3+(1)\right)^9\right]=\left[\sqrt[5]{0}+\left(1+1\right)^9\right]=2^9=512$

6. $\lim_{x\to 2}\left(x^2+1\right)\left(x^2+4x\right)=\left((2)^2+1\right)\left((2)^2+4(2)\right)=(5)(12)=60$

7. $\lim_{w\to -2}\sqrt[3]{\frac{4w+3w^3}{3w+10}}=\sqrt[3]{\frac{4(-2)+3(-2)^3}{3(-2)+10}}=\sqrt[3]{\frac{-8-24}{-6+10}}=\sqrt[3]{\frac{-32}{4}}=\sqrt[3]{-8}=-2$

8. $\lim_{x\to 3}\frac{x^2-3x+12}{x+3}=\frac{(3)^2-3(3)+12}{(3)+3}=\frac{9-9+12}{3+3}=2$

9. $\lim_{x\to -4}|x+4|=|-4+4|=0$

10. $\lim_{x\to 0}\frac{1}{x^2}=$ *undefined*

11. $\lim_{x \to 0} \frac{|x|}{x} = \textit{undefined}$

12. $\lim_{x \to 3} \sqrt[3]{2x^2 - 10} = \sqrt[3]{2(3)^2 - 10} = \sqrt[3]{18 - 10} = \sqrt[3]{8} = 2$

13. $\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = \lim_{x \to 1} \frac{(x-1)(x^2 + x + 1)}{(x-1)} = \lim_{x \to 1} (x^2 + x + 1) = 1^2 + 1 + 1 = 3$

14. $\lim_{x \to 0} \frac{\tan x}{x} = \lim_{x \to 0} \frac{\frac{\sin x}{cosx}}{x} = \lim_{x \to 0} \frac{\sin x}{xcosx} = \lim_{x \to 0} \cdot \frac{\sin x}{x} \cdot \frac{1}{\cos x} = 1 \cdot 1 = 1$

15. $\lim_{x \to 0} \frac{\cos x}{x} = \textit{undefined}$

16. $\lim_{x \to \infty} \frac{1 - \cos x}{x} = \infty$

17. $\lim_{x \to 1} \frac{4 - \sqrt{x}}{x - 16} = \frac{4 - \sqrt{1}}{1 - 16} = \frac{3}{-15} = -\frac{1}{5}$

18. $\lim_{x \to \pi/2} \frac{\cos x}{\cot x} = \lim_{x \to \pi/2} \frac{\cos x}{\frac{\cos x}{\sin x}} = \lim_{x \to \pi/2} \frac{\cos x \sin x}{\cos x} = \lim_{x \to \pi/2} \sin x = 1$

Note $\sin 90° = 1$